

Full Length Research Paper

Stochastic redundancy allocation problem using simulation

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Reliability is one of the main parameters of the products considered to be important to survive in the competitive market. Reliability in its simple form means the probability that a failure may not occur in a given period of time that is, the component performs adequately without failure. The subsystems or components in a system are arranged in series or parallel depending on the space constraint. However, the reliability of the system with components arranged in parallel is more than that system for which the components are arranged in series. Redundancy is the method of arranging the components in a subsystem in parallel such that if one component fails then the other component automatically comes into operation. Through redundancy, any desired level of reliability can be obtained, but in doing so, we have to invest money or other material resources to achieve the desired reliability. A designer has to consider the economic views of the organization in designing a system with high reliability. In this paper, a complex system (series-parallel system) is considered with stochastic reliability for its components. For a particular configuration, reliabilities of the components are generated and evaluated by the system reliability, after which simulation is run and repeated for various runs, before finally consolidating the configuration reliability and resource utilization. The simulation is carried for various feasible configurations and each configuration evaluated. The configuration with best reliability within resource restrictions is selected.

Key words: Redundancy, allocation, reliability.

INTRODUCTION

A lot of work has been reported in series system to its simplicity in configuration. Federowicz and Muzumdar (1968) developed a geometric program to maximize reliability achieved by redundancy. Henley and Gandhi (1975) developed a procedure for generating the reliability function directly from the Boolean algebra transmission function. Tillman and Liittschwager (1967) presented a new method for the optimization of system reliability with linear constraints using parametric Approach.

In parallel configuration, Lijtschwager (1964) used dynamic programming for a solution of a multistage

reliability problem. Banerjee (1976) developed a program to evaluate optimal redundancy allocation for non series – parallel networks. Anderson (1969) explained the theory of reliability of parallel systems with repair and switching. Masatoshi (1977) studied some reliability aspects of system design.

In this paper stochastic reliabilities of the components are considered and Monte Carlo simulation technique is employed to arrive at the optimal solution.

Redundancy is the provision of alternative means or parallel paths in a system for accomplishing a given task that all means must fail before causing the system failure. Application of redundancy in the system design is found in almost all types of systems due to numerous advantages over other methods of improving system reliability. The important ones are:

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- (1) Any desired level of reliability can be achieved.
- (2) Design through redundancy needs comparatively less skill on the part of the designer.
- (3) It provides quick solution.
- (4) This method can be employed in the event of failure of all other methods.

Various forms of redundancy – active (hot) redundancy, standby (cold) redundancy, warm redundancy, component redundancy, system redundancy, hierarchical redundancy etc. can be employed in a system, depending on the feasibility. One has to select a suitable form considering such factors as the type of components, type of systems, reliability requirements, resources Optimization is an act of obtaining the best results under given circumstances. In design, construction and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is to either minimize the effort required or to maximize the required benefit.

In most of the practical problems, decisions have to be made sequentially at different points of time, at different points in space and at different levels. The problems in which decisions are to be made sequentially are called sequential decision problems. Since these decisions are to be made at a number of stages, they are also referred to as multistage decision problems. Simulation programming is a logical extension of mathematical techniques, well studied for the optimization of multistage decision problems.

SIMULATION

Simulation is a quantitative technique developed for studying alternative courses of action by building a model of that system and then conducting a series of repeated trial and error experiments to predict the behaviour of the system over a period of time.

In situations where the mathematical formulation of the problem is not feasible, simulation technique is used by representing reality through a model that will respond in the same manner as in a real-life situation. In simulation, a certain type of mathematical model is formulated which describes the real system's operation. The system is divided into various segments and their inter-relationships with some predictable behavior in terms of probability distributions for each of the possible stages of the system are studied. The simulation experiment is then performed on the model of the system. The following are the reasons for adopting simulation in place of other known mathematical techniques:

- (1) It can handle complex systems that require the modeling of interacting stochastic variants. These are the means for modeling empirical or theoretical distribution of real-world parameters.

- (2) Simulation may be the only method available because it is difficult to observe the actual environment.
- (3) Actual observations of a system may be too expensive.
- (4) There may not be sufficient time to allow the system to operate extensively.
- (5) Simulation enables one to study dynamically systems either in real time, compressed time or expanded time.
- (6) Simulation can be used to experiment with new situations about which we have little or no information, so as to prepare for what may happen.
- (7) Simulation analysis can be performed to verify analytical solutions.
- (8) When new elements are introduced into a system, simulation can be used to anticipate bottlenecks and other problems that may arise in the behaviour of the system.

SOLUTION THROUGH SIMULATION APPROACH

This process starts with assigning 1000 random numbers (0.00 to 0.99) to each of these probability distributions. This is done in tabular column shown in Table 1. The cumulative probabilities and range of random numbers allotted are calculated from the input data. Then a random number between 0.00 and 0.99 is generated for each stage, it is compared with each range of random numbers specified for each stage and reliability, corresponding to the appropriate range of random numbers in which the random number falls, is assigned to the stage as its reliability. Generation of random numbers is done through a probabilistic mechanism. Most computer facilities include a generator of this kind in their software libraries. If there is 'n' number of stages in the system, 'n' random numbers are to be generated and stage reliability of stages R1, R2, R3, Rn is calculated. Then the system reliability is calculated for the given complex network as:

$$R = \{1-[1-(1-r_1^{n_1}) (1-r_2^{n_2}) (1-r_3^{n_3})]\}r_4^{n_4} \dots\dots(1)$$

In addition to reliability considerations, the design of the system may also be constrained by cost, and volume. Let c_i denote the cost of a component i , The total system cost is:

$$C = \sum_{i=1}^m c_i n_i$$

$n_i \geq 1$, integer $i = 1, 2, 3, 4$

The cost of the components N1, N2, N3 and N4 are assumed as 1000, 1200, 1400 and 1600 respectively. The cost of the system is restricted to Rs13,000 and the volume is constrained to the maximum of four identical

components.

The formulation of P_1 is designed to achieve maximum system reliability of the subject C_0 , which is an upper limit on the total system cost.

Therefore, this process is formulated and repeated for different component combinations of the system. If there are 'n' stages and maximum of 'm' components in each stage, the total number of component combinations achieved is m^n . So totally $n \times (m \times n)$ random numbers are generated.

The cost constraint is also taken into consideration. The component combination of the system with highest reliability and satisfying the cost constraint is considered as the optimal solution after first simulation run. Conducting simulation at once may not yield optimal solution, since each simulation run is like a single experiment conducted under a given set of conditions. So for effectiveness, the component combination that has occurred as the optimal solution for more number of times is taken as the final optimal solution after finite simulation runs.

PROBLEM DESCRIPTION

The network consists of components connected as shown in Figure 1 with reliabilities and probabilities as shown in Table 1.

CONCLUSIONS

It is cumbersome to apply optimization techniques or stochastic NLP techniques to find the optimal solution for redundancy design problems in complex circuits. For the given complex network (series- parallel), reliability equation can be written as Equation 1 which is difficult to optimize using dynamic programming approaches and hence Monte Carlo simulation technique is employed in this work and proved to be simplified approach/logic to tackle such problems. For the given network using simulation approach, optimal redundancy is evaluated to be $n_1=1$; $n_2=1$; $n_3=3$; $n_4=4$ by consuming cost resource of Rs 12,800/- and the reliability of the system is found to be 0.9946

%Mention the constraint value

CV=13000;

p = struc(1:4,1:4,1:4,1:4)

for i=1:size(p,1)

n1=p(i,1);

n2=p(i,2);

n3=p(i,3);

n4=p(i,4);

for j=1:500

r11(j)=rand;

r22(j)=rand;

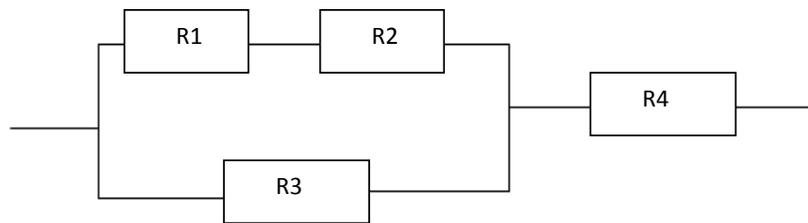
```
r33(j)=rand;
r44(j)=rand;
if (r11(j)<=0.99)&&(r11(j)>=0.7)
    r1(j)=0.75;
elseif (r11(j)<=0.69)&&(r11(j)>=0.3)
    r1(j)=0.7;
else
    r1(j)=0.65;
end
if (r22(j)<=0.99)&&(r22(j)>=0.8)
    r2(j)=0.85;
elseif (r22(j)<=0.79)&&(r22(j)>=0.4)
    r2(j)=0.8;
else
    r2(j)=0.75;
end
if (r33(j)<=0.99)&&(r33(j)>=0.6)
    r3(j)=0.9;
elseif (r33(j)<=0.59)&&(r33(j)>=0.4)
    r3(j)=0.85;
else
    r3(j)=0.8;
end
```

```
if (r44(j)<=0.99)&&(r44(j)>=0.7)
    r4(j)=0.8;
elseif (r44(j)<=0.69)&&(r44(j)>=0.3)
    r4(j)=0.75;
else
    r4(j)=0.7;
end
```

```
end
C1=1000;
C2=1200;
C3=1400;
C4=1600;
TC(j)=n1*C1+n2*C2+n3*C3+n4*C4;
%create a loop to find the satisfying TC for a given n1,
n2, n3, n4
if TC(j)<CV
    b1=(1-(1-r1(j)).^n1);
    b2=(1-(1-r2(j)).^n2);
    b3=(1-(1-r3(j)).^n3);
    b4=(1-(1-r4(j)).^n4);
    value(j)=(1-(1-(b1*b2))^(1-b3))*b4;
else
    value(j)=0;
end
averagevalue=mean(value);
j=j+1;
end
avgvalue(i)=averagevalue;
maxavg=max(avgvalue);
end
%create a loop to find the combination.....
for i=1:256
    if(avgvalue(i)==maxavg)
        z=i;
```

Table 1. Cumulative probabilities and ranges of random numbers allotted.

Component	Reliability	Probability	Random number allocation
1	0.65	0.3	00-29
	0.7	0.4	30-69
	0.75	0.3	70-99
2	0.75	0.4	00-39
	0.8	0.4	40-79
	0.85	0.2	80-99
3	0.8	0.4	00-39
	0.85	0.2	40-59
	0.9	0.4	60-99
4	0.7	0.3	00-29
	0.75	0.4	30-69
	0.8	0.3	70-99

**Figure 1.** Network of components.

```

end
p(z,:);
%create a loop to find the reliabilities to the
corresponding combination
a=p(z,:);
n1=a(1);
n2=a(2);
n3=a(3);
n4=a(4);
for j=1:500
r11(j)=rand;
r22(j)=rand;
r33(j)=rand;
r44(j)=rand;
if (r11(j)<0.99)&&(r11(j)>0.7)
r1(j)=0.75;
elseif (r11(j)<0.69)&&(r11(j)>0.3)
r1(j)=0.7;
else
r1(j)=0.65;
end
if (r22(j)<0.99)&&(r22(j)>0.8)
r2(j)=0.85;
elseif (r22(j)<0.79)&&(r22(j)>0.4)
r2(j)=0.8;
else
r2(j)=0.75;
end
if (r33(j)<0.99)&&(r33(j)>0.6)
r3(j)=0.9;
elseif (r33(j)<0.59)&&(r33(j)>0.4)
r3(j)=0.85;
else
r3(j)=0.8;
end
if (r44(j)<0.99)&&(r44(j)>0.7)
r4(j)=0.8;
elseif (r44(j)<0.69)&&(r44(j)>0.3)
r4(j)=0.75;
else
r4(j)=0.7;
end
C1=1000;
C2=1200;
C3=1400;
C4=1600;
TC(j)=n1*C1+n2*C2+n3*C3+n4*C4;
%create a loop to find the satisfying TC for a given n1,
n2, n3, n4

```

```

if TC(j)<=CV
    b1=(1-(1-r1(j)).^n1);
    b2=(1-(1-r2(j)).^n2);
    b3=(1-(1-r3(j)).^n3);
    b4=(1-(1-r4(j)).^n4);
    value(j)=(1-(1-(b1*b2))*(1-b3))*b4;
else
    value(j)=0;
end
averagevalue=mean(value);
j=j+1;
maxt=max(averagevalue);
end
%create a loop to find the combination.....
p(z,:);
r11=r11';
r22=r22';
r33=r33';
r44=r44';
r1=r1';
r2=r2';
r3=r3';
r4=r4';
value=value';
R=[r11 r1 r22 r2 r33 r3 r44 r4 value];
disp('Cost of the system is.....');
disp(TC(j-1));
disp('Reliability of the system is.....');
disp(averagevalue);

```

```

% disp('number of components of r1 r2 r3 r4....');
disp(' RN1 R1 RN2 R2 RN3 R3
      RN4 R4 System reliability');
disp(R);

```

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